

of the author's achievement of having written a useful book which is also pleasurable to read.

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14[2.05].—GÜNTER MEINARDUS, *Approximation of Functions: Theory and Numerical Methods*, translated by Larry L. Schumaker, Springer-Verlag, New York Inc., 1967, viii + 198 pp., 24 cm. Price \$13.50.

This is a translation of the German edition which appeared in 1964. It differs in detail from that edition by inclusion of new work on comparison theorems for regular Haar systems and on segment approximation.

H. O. K.

15[2.05].—LEOPOLDO NACHBIN, *Elements of Approximation Theory*, D. Van Nostrand Co., Inc., Princeton, N. J., 1967, xii + 119 pp., 20 cm. Price \$2.75.

It is appropriate to begin by pointing out that the subject matter of this book is not *best* approximation; the author is rather concerned with the problem of *arbitrarily good* approximation.

More precisely, the author works within the framework of a given function algebra $C(E)$, consisting of all continuous scalar (real or complex, depending on the circumstances) functions on a completely regular topological space E . Such algebras are given the compact-open topology and the general problem is then to characterize the closure of various subsets S of $C(E)$.

The results given include the following cases: S is a lattice (Kakutani-Stone theorem), an ideal, a subalgebra of $C(E)$ (Stone-Weierstrass), or a convex sublattice (Choquet-Deny). In particular, these results imply criteria for the density of S in $C(E)$.

In addition to these well-known theorems, there is a careful presentation of a general *weighted* approximation problem. This problem is a generalization to the $C(E)$ context of the classical Bernstein problem on R^1 or R^N , and is largely based on recent work by the author. The problem is reduced back to the one-dimensional Bernstein problem and various criteria for its solution are then established, making use of analytic or quasi-analytic functions on R^1 .

The book will be accessible to readers with a modest background in analysis (Taylor and Fourier series, Stirling's formula) and general topology (partition of unity, Urysohn's lemma). The necessary functional analysis of locally convex spaces is developed in the early chapters. The Denjoy-Carleman theorem on quasi-analytic functions is the only other major result needed and references for its proof are provided. There is an extensive bibliography, but no index or exercises.

It is clear that numerical analysts will find material on approximation more relevant to their profession in, for example, the books of Cheney or Rice. On the other hand, Nachbin's book provides an interesting blend of hard and soft analysis, and more importantly, it collects together for the first time the main closure

theorems in function algebras. For these reasons the book represents an important contribution to the mathematical literature. But it also merits additional kudos: the author is noted for (among other things) the clarity of his mathematical exposition and the present book continues in this trend. It is a pleasure to read!

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16[2.05, 7, 12].—JOHN F. HART, E. W. CHENEY, CHARLES L. LAWSON, HANS J. MAEHLY, CHARLES K. MESZTENYI, JOHN R. RICE, HENRY G. THACHER, JR. & CHRISTOPH WITZGALL, *Computer Approximations*, John Wiley & Sons, Inc., New York, 1968, x + 343 pp., 23 cm. Price \$17.50.

This encyclopedic work represents the culmination of the combined efforts of the authors and other contributors to provide an extensive set of useful approximations for computer subroutines, along the lines of the pioneer work of Hastings [1]. The stated aim of the book is "to acquaint the user with methods for designing function subroutines (also called mathematical function routines), and in the case of the most commonly needed functions, to provide him with the necessary tables to do so efficiently."

The book divides into two parts: the first four chapters deal with the problems of computation and approximation of functions in general; the last two chapters (following one devoted to a description and use of the tables) consist, respectively, of summaries of the relevant mathematical and computational properties of the elementary and selected higher transcendental functions and tables of coefficients for least maximum error approximations of appropriate forms for accuracies extending to 25 significant figures.

The wealth of material may be more readily inferred from the following summarization of the contents of the individual chapters and appendices.

The introduction to Chapter 1 (The Design of a Function Subroutine) emphasizes that the efficient computation of a function no longer is simply mathematical in the classical sense, but requires a thorough knowledge of the manner of operation and potentialities of the computing equipment involved. The principal matters considered in this chapter are certain general considerations in preparing a function subroutine, the main types of function subroutine, special programming techniques, subroutine errors, and final steps in preparing a subroutine.

Chapter 2 (General Methods of Computing Functions) summarizes the various techniques for evaluating a function; these include infinite expansions (series, continued fractions, infinite products), recurrence and difference relations, iterative techniques, integral representations, differential equations, polynomial and rational approximations, and transformations for the acceleration of convergence.

Chapter 3 (Least Maximum Approximations) contains a discussion of the characteristic properties of least maximum approximations, together with a description of the second algorithm of Remez for the determination of the optimum polynomial approximation of given degree to a specified function in the sense of Chebyshev. Also considered are nearly least maximum approximations resulting from the